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B R Y N   M A W R

November 27, 2002

Bob Cassanova
Director
NIAC

Dear Bob,

Please accept this, our final report of our Phase I work on the project:
Investigation of the Feasibility of Laser Trapped Mirrors (LTMs) in Space.

Over the last six months we have learned a great deal about the LTM concept and remain optimistic about its potential for making contributions to NASA's programs. You will see many of our initial questions have been answered favorably, some have been rearticulated as we have learned more, and still others remain to be addressed. We are currently working on a Phase II proposal. The collaboration feels strongly about including an experimental component in the Phase II work and this has resulted in spirited debate about the best course of action to undertake. The extended nature of these conversations makes it likely that we will not have a completed Phase II proposal before the December 2nd 2002, deadline. We anticipate however, that we will have a competitive Phase II proposal for the next call in 2003.

It has been a pleasure to have this opportunity. Thank you again for your support.

With best regards,

Elizabeth McCormack

Investigation of the Feasibility of Laser Trapped Mirrors in Space
Phase I: Final Report

I. Introduction

The Laser Trapped Mirror (LTM) was first proposed by Antoine Labeyrie in 1979 as an innovative approach to the production of large, lightweight optics in space. Labeyrie's idea was to use counter-propagating laser beams to create large parabolic, interference fringes. By tuning the laser wavelength in single mode from red to blue in successive saw-tooth steps, the particles could be trapped and collected into a central fringe, yielding a single, parabolic sheet of material. The result of this process would be a reflective surface of almost arbitrary size, which could serve as the primary of a large telescope. Remarkably, a 100-nm thick, 35-meter diameter mirror would only require about 100 grams of material.

In Phase 1 we explored, principally through modeling, Labeyrie's original vision and variants of it. The goal was to determine whether the LTM could be a practical solution to the problem of building large, low mass optical systems in space. We can say from the outset that while there are considerable technical obstacles in the way of constructing an LTM and much of the physics involved is just beginning to be investigated, we have not found any physical barriers to creating the LTM.

The principal challenge in both constructing and maintaining such a mirror is due to the small trapping forces associated with laser light. If we consider two colliding plane waves, the Poynting flux shows that a standing wave is created with a trap potential energy depth of

$$E_{Trap} = \frac{2\pi\alpha}{c} I ,$$

where α is the polarizability of the particle trapped in the well and I is the laser intensity. If we take for our typical particles those with radius, a = 1 micron, then for common dielectric indices of refraction, $\alpha \sim 0.3\, a^3 \sim 0.3 \times 10^{-12}$ cm^3, the trap depth is $E_{trap} \sim 6 \times 10^{-20}\, I$ ergs when I is expressed in Watts/m^2. This is an extremely small number. In terms of equivalent temperature, it is of order mK, or in terms of escape velocity of a 1-micron particle from the trap it is of order 10^{-4} cm/s. For comparison, the galaxy is pervaded by an infrared background of T ~ 30 K. Thus, if the mirror is allowed to equilibrate with this background energy bath, the particles will be moving at speeds far higher than the trap escape velocity and the mirror will evaporate.

II. Estimate of Evaporation Time

A simple estimate of the evaporation time can be made as follows. Under a collision with the infrared background photons, a nanosphere of mass m will pick up a kinetic energy $\Delta E = p_\gamma^2/2m = (h/\lambda)^2/(2m)$ resulting in an energy absorption rate of

$$\frac{dE}{dt} = \Delta E n_\gamma \sigma c \,.$$

The appropriate cross section σ is the extinction cross section which, for the wavelengths corresponding to 30K, is approximately $10^{-2}\, \sigma_{\text{geometric}} = 10^{-2}\, \pi\, a^2$[1]. Since everything on the right is a constant, the equation can be immediately integrated and E set equal to E_{Trap}. This gives the evaporation time of the mirror as

$$\tau_{evap} = \frac{4\pi\alpha m \lambda^2}{h^2 n_\gamma \sigma c^2} I \,.$$

Assuming a density of photons that corresponds to the blackbody peak of 30K ($n_\gamma \sim 4 \times 10^6$; $\lambda = 10^{-2}$cm), and a mass density for the particles of $\rho = 1$ g/cm^3, we find for 1 micron-sized particles,

$$\tau_{evap} \approx 1.5 \times 10^8\, I(W/m^2)\,\text{sec}\,.$$

This is a respectable number, about five years for I=1 W/m^2, and on the order of months for currently available laser intensities. According to astronomical data for galactic emissions in the ultraviolet to infrared range [2], the above estimate for n_γ may be as much as an order of magnitude too large, and we may also be able to add two orders of magnitude due to the optical binding potential (discussed below), which would make the mirror lifetime essentially infinite without the need for cooling mechanisms. However, note that the evaporation time scales as the radius to the fourth power, a^4. If we go to smaller particles, for instance 100-nm particles, we immediately lose four orders of magnitude, putting the evaporation time on the timescale of hours. For this reason particle size will be a critical factor in determining the rate at which energy may need to be removed from the mirror. We return to this issue below.

III. Dynamical Simulations

One would like to confirm the above estimate of evaporation time with dynamical calculations. To do this we developed a single-particle, 1-D model, in which a particle in the laser trap is bombarded by photons of a fixed momentum coming in randomly from either the left or right. We soon realized that this model was not realistic and of limited use because it exhibits chaotic behavior. In retrospect, this result is not surprising. The trap potential is of the form $V \sim \sin^2(kx)$, which leads to a nonlinear force term $f \sim \sin(kx)\cos(kx)$. Also, the forcing term due to the photon collisions is inherently stochastic. Such characteristics of the equations of motion are known to produce chaotic dynamics. We were able to roughly confirm the analytical estimate for low laser intensities (< .01 W/m^2) to within a factor of four, but the detailed results were unpredictable and computational time limitations precluded going to higher intensities.

In any case, the single particle model is not realistic. One needs to take into account the optical binding potential, as well as any damping mechanisms, which are sure to dominate over the single-particle behavior. This requires at least a 2-D model with interactions computed between all particle pairs.

IV. Optical Binding Potential

Trapping the dielectric spheres in the laser potential causes dipole moments to be induced in the spheres, which in turn gives rise to electromagnetic forces between the spheres. Burns et al. [3] give an approximation for the potential between two such dipoles. It is a 1/r potential with oscillations periodic in the trapping laser wavelength and is highly anisotropic, being almost zero when the line connecting the oscillators is parallel to the light polarization axis and much larger in the orthogonal direction. Using their expression for a laser wavelength equal to the particle radius, one can compute the potential experienced by a particle at the origin due to its nearest neighbors. Assuming about 20 nearest neighbors per quadrant one gets that the potential is roughly 300 times the trap depth. This would be the potential experienced by any particle in the mirror not near the edge and accounts for the two orders of magnitude gained in mirror lifetime mentioned earlier. Clearly this effect will be extremely significant, if not crucial to the design of the LTM.

Several important points, however, need to be examined about this result. The first is that the approximations made if Ref. 3 assume that the dipole separation is greater than a wavelength and that the wavelength is greater than the particle size. Consequently, their approximations break down the regime of interest to us, where $\lambda_{laser} = a$, the particle radius, the Mie scattering regime. An analytical solution to the problem in this regime does not exist so numerical approaches are required. Tony Rothman and Peter Anninos have worked to develop a numerical code to calculate the interaction potential between multiple dipoles. At this time, work on the code is in process and the preliminary results are encouraging; they agree with those of Burns et al. [3] in the regime where the approximations are valid and indicate that there is indeed a significant optical binding effect. At the moment, however, we cannot give an exact value for the binding potential due to outstanding issues concerning how to properly scale the absolute magnitude of the interactions. Furthermore, numerous diagnostics should also be performed, as well as an exploration of the full parameter space (wavelength vs. particle size and separation). These numerical calculations will be important area to pursue in Phase II of the project.

One can remove the angular dependence of the interactions by rotating the axis of polarization of the laser light faster than the dynamical timescale (which is on the order of one second). This averaging will smooth out the potential. A related question that Jean-Marc Fournier has raised is whether photons from the laser itself will significantly heat the nanospheres. This question was discussed at a meeting in France at the Observatoire Haute Province (OHP) and the general consensus was that the cross section can be made low, and thus such heating can be avoided. However, the question again is one of timescales. Even low absorption will lead to mirror evaporation on some timescale unless there is some cooling or damping to counter its small but nonzero associated heating rate.

V. Damping Mechanisms

If one wants to go to a particle size much smaller than one micron, damping (cooling) mechanisms will be required to keep the particles in the trap. Our original thoughts included investigating two techniques used with success in cooling atoms and particles in liquids; Doppler cooling and collisional damping. On closer inspection, however, it seems that neither will prove particularly effective for our application.

Doppler cooling depends on sharp resonant absorption lines in the sample. The cooling timescale is given by $\tau_{cool} \sim m/Q$, where m is the particle mass and Q, the Q-factor of the resonance, is defined as $\lambda/\Delta\lambda$. Almost all Doppler cooling experiments to date rely on atoms of a given element, which by definition are identical. Hence the absorption lines are all identically narrow, resulting in $Q \sim 10^8$. Mie absorption lines do exist for nanospheres approximately one wavelength in diameter. Unfortunately, in a realistic sample of spheres, one expects a variation of properties, resulting in a smeared out absorption spectrum, perhaps lowering the Q to ~ 100 [4], so we may be up by a factor of as much as 10^{16} from atomic systems in the time needed to cool. The timescale for cooling then becomes comparable to or exceeds the estimate of the evaporation time of the mirror. If we were to use a Mie resonance, to keep the cooling timescale short one would need a very pure sample of nanospheres. Manufacturing techniques of nanospheres however, are in their infancy and it is difficult to estimate what percentage of particles would need to be discarded to get sufficiently uniform sample parameters.

At the OHP meeting, Antoine Labeyrie suggested that lining up many spheres would narrow the absorption of the system as a whole and thus provide more effective Doppler damping. This effect remains to be investigated, however, if this is indeed the case, it should emerge in a realistic model of the detailed electromagnetic interaction among particles. This will be something to look for in the results from the multiple particle model being developed by Rothman and Anninos.

Collisional damping does not appear to be an effective mechanism to pursue either. Presumably collisions between two nanospheres in the well can be made slightly inelastic. Thus, kinetic energy could be converted into internal modes. For damping to take place, however, the heat generated must be radiated to the exterior, i.e. coupled to the space environment. We have already estimated the heating rate for a nanoparticle in a thermal bath of infrared photons, and it is rather slow. On very general principles of reciprocity, we therefore expect that collisional damping will be similarly ineffective. Because these considerations are completely general, they apply to any mechanism relying on converting excess energy into infrared photons. For example, we debated coating the nanospheres with glue or attaching lossy springs or DNA tails to them to damp center-of mass-motion. However, unless one can radiate this energy into space, the energy has not been removed.

These somewhat disappointing assessments have led us to consider several others, more sophisticated damping mechanisms. Several mechanisms relying on collective effects in the system of particles were suggested in conversations with Robin Kaiser and colleagues. One example discussed was three-body collisional, or evaporative cooling. When three bodies interact, (or equivalently, two bodies in a potential well), one body may be given enough energy to escape leaving the remaining body with less energy. This technique has been highly effective in creating Bose-Einstein condensates in atomic vapors. With respect to the LTM one would need to investigate numerically the number and density of particles needed initially to achieve a significant remaining number at a given temperature as well as the time this cooling mechanism would require.

Stochastic cooling is another potentially effective mechanism. If, for instance, one knows the mean momentum of a cloud of particles, one can apply a kick to center the momentum distribution on zero, using for example, adaptive optics, thus cooling the cloud. Such a technique has been used at CERN to cool antiprotons, which ultimately led to the discovery of the W and Z bosons. Raizen et al. have recently proposed a way of extending this technique to laser cooling of atomic systems [5]. To implement stochastic cooling in the LTM would require sampling the momentum of the mirror particles at a rate faster than the dynamical time scale, and then correct for it. As the dynamical timescale is rather long, on the order of one second, this should be possible. The quality of the diffraction pattern could be used as a measure of the momentum distribution of the particles and so it may be that monitoring the diffraction pattern produced by the mirror could be the simplest method for implementing stochastic cooling.

In a related vein Zajfman et al. have recently discovered that ions in an anharmonic potential well tend to bunch rather than to diffuse, even if the particles are of the same charge [6]. This is due to the fact than in anharmonic oscillator potentials, the period is velocity dependent and faster particles catch up with slower ones, tending to clump together. It is quite conceivable that the optical binding potential will exhibit similar collective effects. If, for example, it turns out that particles do tend to bunch in velocity space, then this center-of-mass motion could easily be removed by adaptive optics, which would amount to cooling. In general, there is presently no reason to think exploiting these effects would not work; however the physics involved is just now becoming an active area of study.

Finally, a very attractive option to explore is to use *endohedrals*, that is, molecular structures consisting of atoms confined within fullerene cages for the mirror particles instead of solid spheres. Atoms trapped in a fullerene cage can exhibit sharp resonant absorption features and the cage itself can have wideband reflection properties. Examples of such systems include encapsulated rare earth ions as well as caged scandium atoms, e.g., $Sc_3N@C_{84}$ [7,8]. With these systems one can imagine taking advantage of the sharp absorption lines to implement Doppler cooling and use the wide-band properties of the fullerene for the mirror response.

VI. Image Quality

As emphasized earlier, the mirror evaporation time is an extremely sensitive function of particle radius. Clearly, using larger particles is much more advantageous from the point of view of maintaining the mirror. However, because the LTM is in effect an enormous two-dimensional diffraction grating, the question arises as to what effect particle size has on the image quality. The consensus at the OHP meeting was that the resolving power of the mirror should be the same as an ordinary mirror (approximately λ/aperture size) as long as the observing wavelength is at least twice the particle diameter, and could be approximately $\lambda/100$ with adaptive optics. It is certainly true that in the long-wavelength regime, the telescope will act like an ordinary mirror.

The effect of particle size on the mirror efficiency will depend on the amount of scattering in the backward direction from each particle in the LTM. This in turn will depend on the ratio λ/a and the number of scatterers. In the Mie regime, where $\lambda \sim a$, calculations must be performed numerically. Codes being developed at the Fresnel Institute in Marseilles by Daniel Maystre should help determine the efficiency of the mirror. In the meantime, preliminary results with a code we have written indicate that, with a small number of particles, of order one hundred, reflection efficiencies ranged between 10^{-4} and 10^{-2}, depending on particle size. This result could change substantially when, more realistically, an extremely large number of particles is considered.

VII. Trap Loading

An additional serious challenge we face is devising a way to initially make the LTM, that is, to load the laser trap with very cold nanoparticles. We have discussed two possible methods, both of which need to be further explored. The first would be simply to freeze the nanoparticles to a large sheet or web of, for example, Mylar at liquid helium temperatures and let the particles evaporate to be captured by the trapping light. The success of this scheme depends crucially on the rate at which the particles evaporate from the sheet, which in turn depends on the "work function" of the material. An alternative version of this scheme would be to use piezoelectric vibration to gently shake the particles loose from a substrate. We can also imagine turning this around and using a substrate that would evaporate, leaving behind the very cold nanoparticles. For example, by embedding nanoparticles in frozen liquid helium, as the helium evaporates the nanoparticles would be left behind with extremely low velocities.

Alternatively, taking an entirely different approach, one might build up the mirror with a small "edge-rover" spacecraft housing two counter-propagating laser beams to hold and deliver the particles to the central fringe of the mirror. The particles could be placed into the mirror light trap with negligible transverse velocity and a small horizontal velocity controlled by a diverging lens that would produce a backward retarding force, as in optical tweezers. The rover would fill the mirror trap continuously by flying a spiral pattern outward. Necessarily, the time to construct the mirror would need to be less than the evaporation time of the LTM.

VIII. Conclusions

The LTM is a potentially revolutionary approach to constructing and maintaining large telescopes in space, however many questions surrounding the concept need to be addressed before a commitment to serious development work can be made. In our Phase I effort, we have explored several key parameters to determine the power requirements and stability characteristics of an LTM. We have three general results to report:

1) A range of particle sizes is feasible at reasonable laser powers although an important tradeoff is involved: larger particles result in stronger trapping forces and require no additional cooling to maintain the LTM, on the other hand, smaller particles, which would require cooling, lead to improved image quality in the visible range of imaging wavelengths.

2) Preliminary numerical modeling results indicate that collective optical binding effects will be very important to the design of the LTM with the potential to provide both additional stabilizing interactions and novel collective cooling mechanisms.

3) Perhaps the most pressing implementation issue that has emerged from our Phase I work is the need to find a viable way to initially load the LTM with sufficiently cold particles.

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